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OPTIMIZATION OF A LARGE NUMBER OF COOLANT PASSAGES  
LOCATED CLOSE TO THE SURFACE OF A TURBINE BLADE

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**ABSTRACT**

A constrained optimization of locations and discrete radii of a large number of small circular cross-section straight-through coolant flow passages in internally cooled gas turbine blade was developed. The objective of the optimization was minimization of the integrated surface heat flux penetrating the airfoil thus indirectly minimizing the amount of coolant needed for the removal of this heat. Constraints were that the maximum temperature of any point in the blade is less than the maximum specified value and that the distances between any two holes or between any hole and the airfoil surface are greater than the minimum specified value. Configurations with maximum of 13 passages and 30 passages were considered. The presence of external hot gas and internal coolant was approximated by using convection boundary conditions for the heat conduction analysis. A parallel three-dimensional thermoelasticity finite element analysis (FEA) code from the ADVENTURE project at University of Tokyo was used to perform automatic thermal analysis of different blade configurations. A robust semi-stochastic constrained optimizer and a parallel genetic algorithm (PGA) were used to solve this problem using an inexpensive distributed memory parallel computer.

**INTRODUCTION**

With a perpetual goal of increasing thermodynamic efficiency of turbines, various blade cooling schemes have been

used. However, with the extremely high temperatures of the combustion gases it became apparent that film cooling causes increased production of NO<sub>x</sub>. As a remedy, a high-pressure closed-circuit internal cooling concept [1] became attractive again after decades of its inception. Moreover, circular cross-section straight-through coolant passages became attractive because of the ease of their manufacturing thus lower cost of such blades. An intuitive approach became to place a large number of such passages very close to the blade outer (hot) surface [2] thus mimicking some of the natural cooling networks appearing in biology. However, the problem that has not been answered yet is where precisely to locate each such coolant passage and what should be the radius of each individual passage.

The design problem considered in this paper involves the parametric shape and topology optimization of coolant passages in a turbine stator blade. We consider configurations that have many small coolant passages that are close to the surface of the blade. All the passage cross-sections are circular. During the design optimization process the radius of each passage may vary within a specified range, or the passage can be removed completely when its optimized radius reduces below a specified minimum value.

The objective is to reduce the amount of heat transferred through the blade surface subject to a maximum blade temperature. This objective indirectly minimizes the amount of

coolant required to maintain an allowable temperature in the blade material. Heat conduction within the blade material is used to compute the objective and the temperature constraint. Instead of a fully three-dimensional conjugate heat transfer analysis [3] or a quasi three-dimensional conjugate heat transfer analysis [4] of each candidate blade configuration, heat convection boundary conditions are used to simulate the presence of coolant and hot gas. This is a very approximate treatment of the fluid mechanics [5], an accurate treatment of heat conduction, and excludes treatment of elasticity concerns [6]. However, it is sufficient for demonstrating the challenging nature of the design problem and the capabilities of the optimization algorithms used to solve the problem.

The main difficulty in this design problem is due to the large number of design variables and the fact that the design variables for radius are mixed discrete/continuous. Thus, the design variables for passage radius create discontinuities in the objective function space methods based on gradient search. Consequently, methods such as the well-known sequential quadratic programming (SQP) are not appropriate for this problem. Also, the nature of the parameterization of this problem leads to many local minima in the objective function space, which creates difficulties for most optimization algorithms. In this paper we present the results of the application of two global optimization methods that do not require gradients and that are robust against the presence of local minima.

Global search methods will typically require hundreds or thousands of design analyses in attempt to find the global solution [7]. In order to complete the design process in a reasonable amount of time, a parallel computer should be employed. Both the finite element analysis and the optimization codes used in this work were written to make full use of parallel computing resources. The global optimization methods used here are naturally parallel algorithms and can make full use of parallelism.

## OPTIMIZATION METHOD

The core of the passage design system is the optimization code. The optimizer directs the design process by generating new designs based on the performance of previous designs, in an iterative manner. In general, we wish to use optimization methods that are robust and efficient. For optimization on a parallel computer, the optimizer should find a good design in the minimum possible number of iterations. Such algorithms should also be capable of making full use of large-scale parallel computers. Since each design analysis is a full 3-D simulation, the total computation time can be from weeks to months if an efficient and sufficiently parallel algorithm is not used.

We also desire a robust optimization algorithm. The optimization process should not terminate in a local minimum and it should not terminate if the analysis cannot be completed occasionally due to, for example, failure to generate a proper grid for a candidate design. For the particular problem considered in this paper, the method should not require gradients of the objective or constraints so that discontinuous design variables can be used (for example, total number of cooling passages which could vary during the optimization).

In our experience, parallel genetic algorithm (PGA) variations [7,8,9,10,] and response surface methods based on Indirect Optimization based on Self Organization (IOSO) [11,12] work well for 3-D turbine coolant passage design optimization.

## IOSO Method

The IOSO method is a constrained optimization algorithm based on response surface methods and evolutionary simulation principles. Each iteration of IOSO consists of two steps. The first step is creation of an approximation of the objective function(s). Each iteration in this step represents a decomposition of an initial approximation function into a set of simple approximation functions. The final response function is a multilevel graph. The second step is the optimization of this approximation function. This approach allows for self-corrections of the structure and the parameters of the response surface approximation. The distinctive feature of this approach is an extremely low number of trial points to initialize the algorithm (30-50 points for the optimization problems with nearly 100 design variables). The obtained response functions are used in the procedures of multilevel optimization with the adaptive changing of the simulation level within the frameworks of both single and multiple disciplines of the object analysis. During each iteration of the IOSO, the optimization of the response function is carried out within the current search area. This step is followed by the direct call to the mathematical model for the obtained point. The information concerning the behavior of the objective function nearby the extremum is stored, and the response function is made more accurate just for this search area. For a basic parallel IOSO algorithm, the following steps are carried out:

1. Generate a group of designs based on a design of experiments method;
2. Evaluate the designs in parallel with the analysis code;
3. Build initial approximation based on the group of evaluated designs;
4. Use stochastic optimization method to find the minimum of the approximation;
5. Do adaptive selection of current extremum search area;
6. Generate a new set of designs in current extremum search area using DOE;
7. Evaluate the new set of designs in parallel with the analysis code;
8. Update the approximation with newly obtained result;
9. Go to 4. unless termination criteria is met.

Thus, during each iteration, a series of approximation functions is built for a particular optimization criterion. These functions differ from each other according to both structure and definition range. The subsequent optimization of the given approximation functions allows us to determine a set of vectors of optimized variables, which are used to develop further optimization criteria on a parallel computer.

## Multilevel Parallelism in Optimization

The usual approach to parallel optimization is to run a single analysis on each processor per optimization iteration. However, a mesh for a geometrically complex design may be

large; sometimes the finite element analysis requires more memory than is available on a single processor. For this reason, the finite element analysis must be distributed among several processors. If a large number of processors are available, we can use all of them by running several simultaneous parallel analyses to evaluate several candidate design configurations. We have developed an optimization communication module with the MPI library [13] that utilizes this multilevel hierarchy of parallelism. This module can be used with any parallel optimization method including PGA and IOSO algorithms. A graphical depiction of the hierarchy of parallelism is shown in Figure 1.

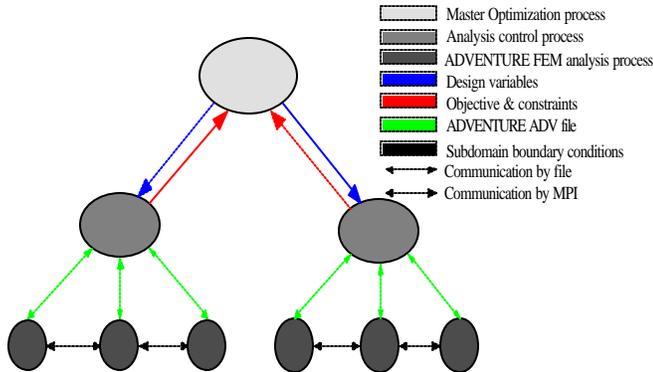


Figure 1: Multilevel parallelism in optimization.

## DESIGN ANALYSIS

The thermal and analysis was performed by parallelized finite element analysis. The finite element analysis codes and tools for mesh generation, mesh partitioning, and others (Fig. 2) are freely available as a part of the ADVENTURE project [14] lead by the University of Tokyo. The finite element solvers are geared towards large-scale parallel analysis and are well suited to the efficient analysis of complicated geometries.

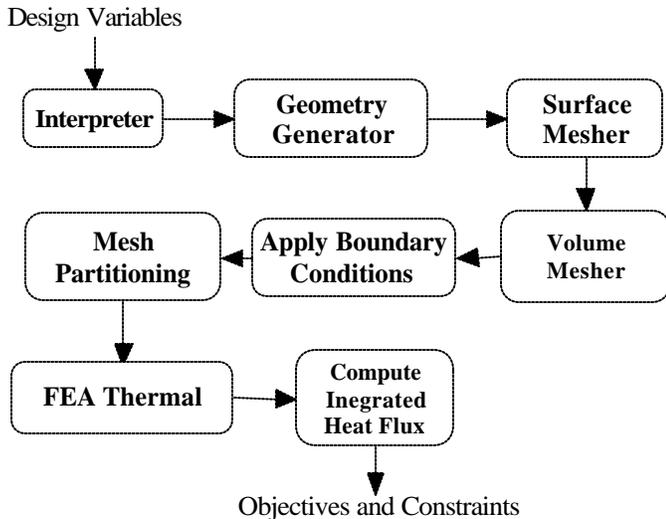


Figure 2: Modules used for automatic parallel FEA.

For each design a series of modules is required to turn given set of design variables into an object and constraint function values. The flow of data between these modules is depicted graphically in Figure 3. The analysis process may need to be performed hundreds or thousands of times for a single optimization run so it is critical that each module be automatic, robust, and computationally efficient.

## OBJECTIVE AND CONSTRAINTS

In this section the design objective and constraint functions are discussed. The objective is to minimize the total amount of heat transferred to the blade (integrated heat flux on the hot surface of the blade) while maintaining a maximum temperature,  $T_{max}$ , which is lower than the maximum allowable temperature,  $T_{allow}$ . This objective indirectly minimizes the amount of coolant required to cool the blade. The minimization of this objective could result in the reduction of the number of cooling passages as well.

The objective function is computed by integrating heat flux across the blade outer surface,  $\Gamma$ . Mathematically, the objective function  $F$  is expressed as

$$F = \int_{\Gamma} k \frac{\partial T}{\partial n} dx dy dz \quad (1)$$

where  $T$  is the blade temperature,  $n$  is the direction normal to the surface  $\Gamma$ , and the constant  $k$  is the heat conduction coefficient for the blade material. There are two inequality constraints that are expressed as

$$G_1 = \frac{T_{allow} - T_{max}}{T_{allow}} \quad (2)$$

$$G_2 = \sum_{i=1}^{nholes} C_i \quad (3)$$

where  $nholes$  is the number of passages and  $C_i$  is a positive number when the distance between passage  $i$  and another passage is less than a specified distance. Otherwise the value of  $C_i$  is zero. The first constraint is necessary so that the maximum temperature in the blade material is always below the maximum allowed temperature. The second constraint is needed to insure that the optimizer only searches for valid geometries. The constraints are satisfied if  $G_1 \leq 0.0$  and  $G_2 \leq 0.0$ .

## DESIGN PARAMETERIZATION

The outer blade shape is considered to be fixed and to be provided by the user at the beginning of the design optimization. Presumably, this is the blade shape that has already been optimized for its aerodynamic performance [15]. The design variables include the radius of each circular passage,  $r_i$ , and position of the passage center,  $\langle x_i, y_i \rangle$ , in the blade cross-section. The passage center is allowed to move normal to the outer contour within a specific region as shown in Figure 3. The design variable  $x_i$  is a distance in the direction normal to the blade surface and is non-dimensionalized so that

it always lies between the two dashed lines shown in Figure 4. The variable  $y_i$  is a non-dimensional distance in a surface following coordinate direction that is taken along the outer surface of the blade.

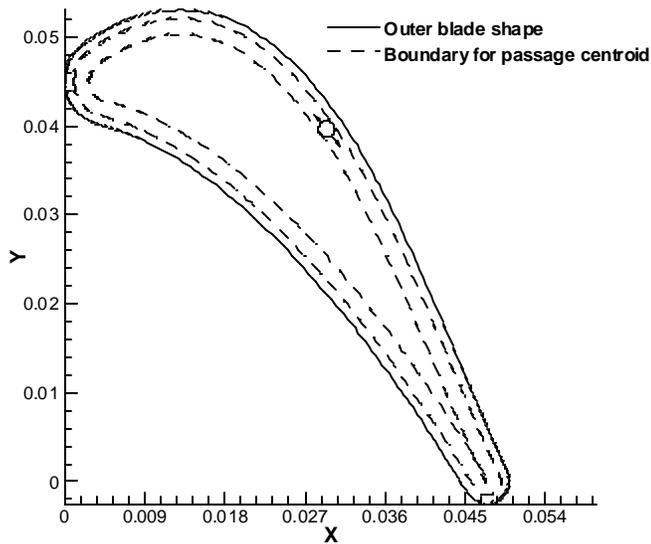


Figure 3. Region where coolant passage centers are allowed.

For 13 and 30 passages, this parameterization leads to a total of 39 and 90 variables, respectively. The passage radius,  $r_i$ , is set to zero if it goes below a specified value,  $r_{min}$ , thereby allowing the optimizer to reduce the total number of passages.

$$r_i = \begin{cases} 0 & r_i < r_{min} \\ r_i & r_i \geq r_{min} \end{cases} \quad (4)$$

This mixed continuous/discrete behavior creates a discontinuity in the objective function space and makes the problem difficult for classical optimization algorithms to solve.

A triangular surface mesh [16] and a tetrahedral volume mesh [17] were generated automatically for each candidate design. The mesh generator did an adequate job of placing enough points between the passages and the blade surface, even when the passages were very close to the surface. Example meshes as shown in Figures 4 and 5. A typical mesh had around 50,000 nodes.

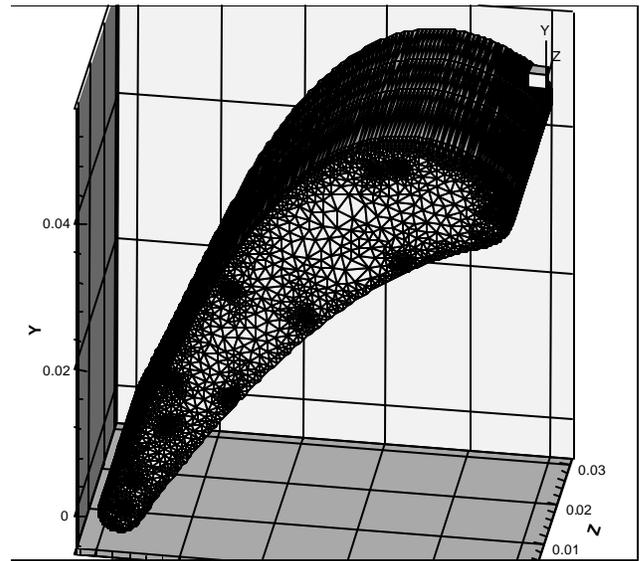


Figure 4: View of a surface mesh.

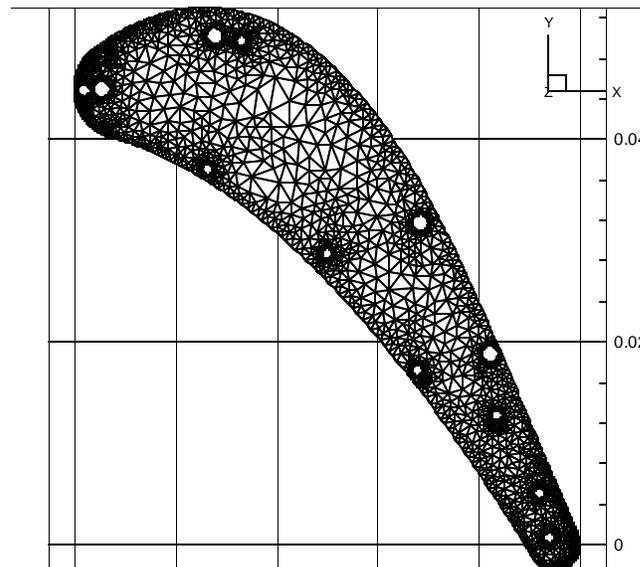


Figure 5: View of mesh on a blade cross-section.

## DESIGN OPTIMIZATION EXAMPLES

In this section, some examples of design optimization using both GA and IOSO optimization methods are presented. For all cases the design variable bounds were set according to Table 1. Additional constants used for all examples are shown in Table 2.

Table 1: Design variable bounds

Parameter	Lower bound	Upper bound
$r_i$	0.25 mm	0.8 mm
$x_i$	1.0 mm	2.75 mm
$y_i$	$\frac{(i-1)}{nholes}$	$\frac{i}{nholes}$

Table 2: Constants used for design examples

Maximum allowable temperature, $T_{allow}$	800.0 °C
Blade heat conduction coefficient, $k$	7.0 W/m <sup>2</sup> °C
Minimum passage radius, $r_{min}$	0.5 mm
Blade axial chord length	5.0 cm
Minimum allowable distance between passages	0.1 mm

We created the outer blade geometry by generating a series of 2-D turbine airfoils [10] and stacking the sections along the blade spanwise direction. Though the generated geometry is not an actual stator blade, we tried to make an outer surface that maintains the characteristic shape of a typical turbine stator. However, if a real outer shape is available from the user, it should be possible to use it directly with the design system with no modifications.

Thermally insulated conditions were used on the blade end surfaces. Convective heat transfer (Robin type) boundary conditions were used on the surfaces of the coolant passages and on the outer blade surface.

Two design cases were run. In the first case the maximum number of coolant passages was set to 13. The total number of design variables was 39. Table 3 shows the values used for the convection boundary conditions for this case.

Table 3: Parameters for initially 13-passage optimization.

Coolant bulk temperature, $T_C$	300 °C
Coolant convective heat transfer coefficient, $h_C$	1000 W/m <sup>2</sup> °C
Hot gas bulk temperature, $T_B$	1500 °C
Hot gas convective heat transfer coefficient, $h_B$	100 W/m <sup>2</sup> °C

This optimization problem was solved using both PGA and IOSO algorithms. Both optimization algorithms were run on 54 processors in a PC cluster composed of Pentium II and Pentium III processors. For both GA and IOSO methods, 28 simultaneous analyses were run per iteration. That is, the design population size was 28. Each finite element heat conduction analysis used 2 processors. The same initial design was given to both optimizers at the start of each run.

The following parameters were used for PGA method: 8.0 percent mutation rate, 50.0 percent chance of uniform crossover, binary encoded variables with 5 bits for  $r_i$ , 4 bits for  $x_i$ , and 8 bits for  $y_i$ . The constraints were enforced by an exterior penalty method. The parameters were chosen based on the primary author's past experiences with solving shape optimization problems by PGA method.

The IOSO method requires only a single tunable parameter. That parameter controls the depth of the global search. For this case the parameter was set for an extensive global search, which is most appropriate for objective functions known to have many local minima.

Figure 6 shows the convergence history for both optimization algorithms. The IOSO algorithm obtained a better design with fewer analyses.

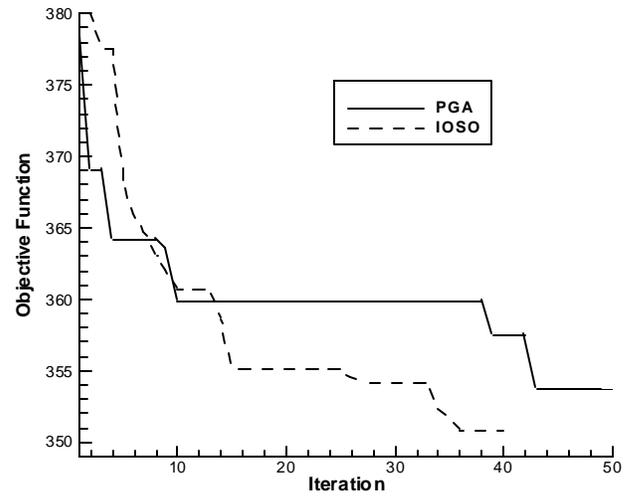


Figure 6: Objective function convergence history.

The convergence criteria for the IOSO method were met by iteration 40 and the process was terminated. The IOSO and PGA best designs both reduced the number of passages from 13 to 8, although the objective function value for IOSO best design was lower as shown in Table 4. However, both best designs represent significant improvements over the initial design.

Table 4: Optimization results for initially 13-passage case.

Result	Initial guess	PGA best design	IOSO best design
$N_{holes}$	13	8	8
$T_{max}$	734.5 °C	785.7 °C	799.9 °C
$F$	432.6 W	353.7 W	350.8 W

Figure 7 shows the passages' configuration for the initial design, the best design from PGA, and the best design from IOSO. The passages' topology in both optimized results are similar indicating that both optimizers appear to be converging to the same design. The PGA optimization was run for an additional 50 iterations and no further improvement was found. The PGA would likely need a larger population and further iterations to reach the same design found by IOSO.

The outer surface temperature on the optimized design is much closer to  $T_{allow}$  than in the initial design as shown in Figure 8. The best possible design is achieved when the entire outer surface temperature is equal to  $T_{allow}$ . It should be noted that in that case, the smallest possible integrated heat flux is 330.3 Watts. However, due to the limitations of the geometric parameterization, such a perfect design is not achievable.

The second design case involved a maximum of 30 cooling passages. The boundary condition parameters for this case are shown in Table 5. This problem was solved using both PGA and IOSO algorithm. For both PGA and IOSO method, 40 simultaneous analyses were run per iteration. Each finite element heat conduction analysis used 2 processors. The same initial design was given to both optimizers at the start of each run. The following PGA parameters were used: 5.0 percent mutation rate, 50.0 percent chance of uniform crossover, and 5 bit binary encoding for all variables.

Table 5: Parameters for initially 30-passage optimization.

Coolant bulk temperature, $T_C$	300 °C
Coolant convection heat transfer coefficient, $h_C$	1000 W/m <sup>2</sup> -°C
Hot gas bulk temperature, $T_B$	3000 °C
Hot gas convection heat transfer coefficient, $h_B$	100 W/m <sup>2</sup> -°C

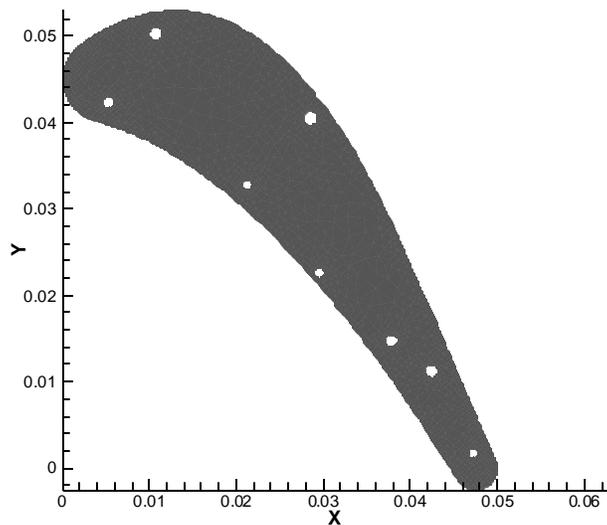
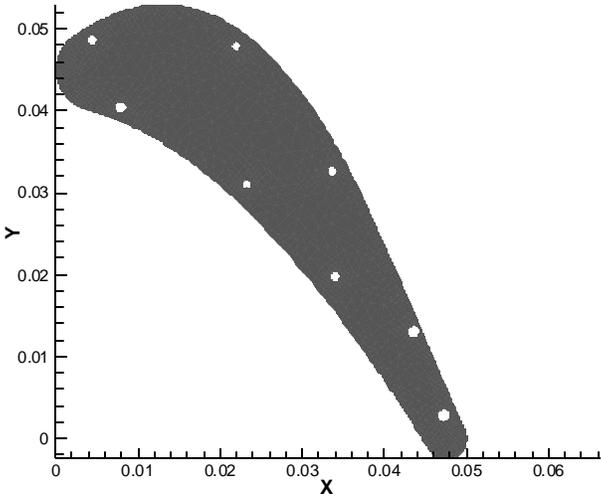
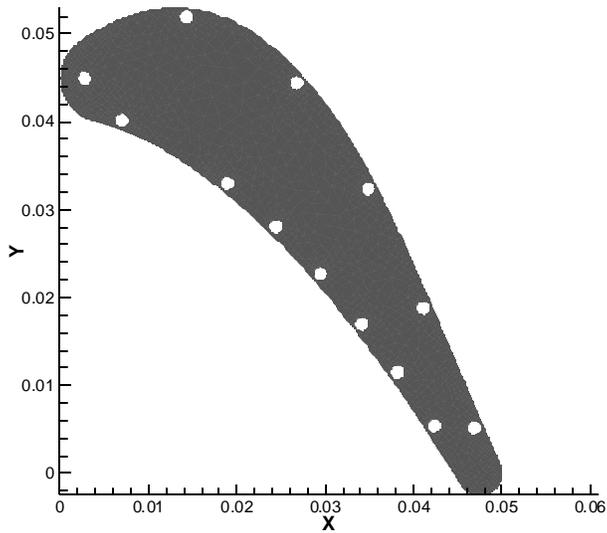


Figure 7: Cooling passages for initial design, PGA best design, and IOSO best design for initially 13-passage case.

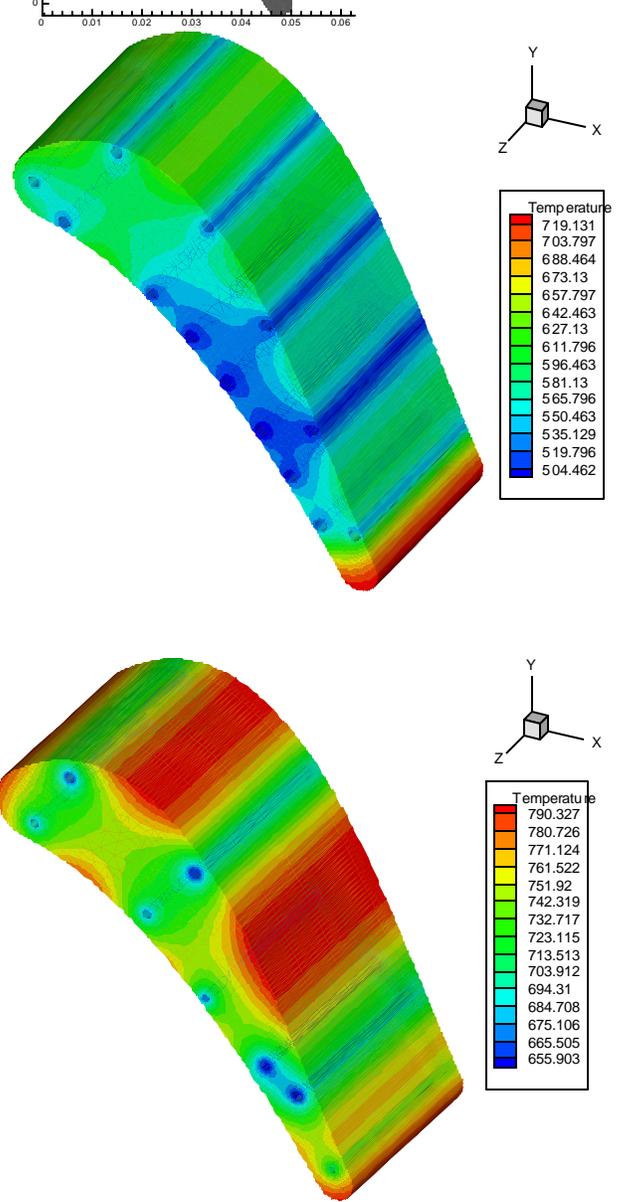


Figure 8: Temperature distribution for initial design and IOSO optimized best design for initially 13-passage case.

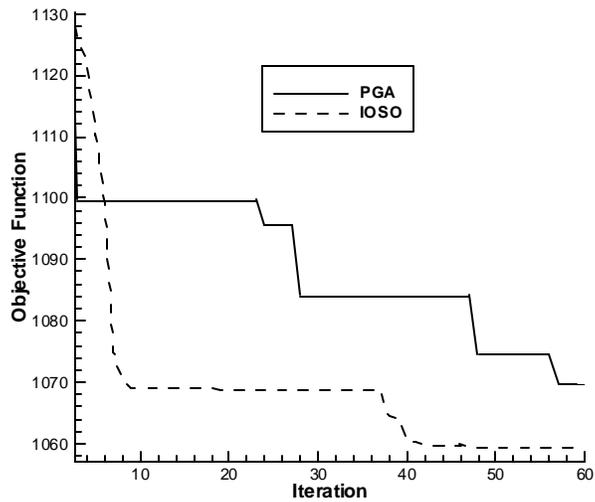


Figure 9: Objective function convergence history for initially 30-passage case.

The convergence history in Figure 9 shows that for the case of 30 cooling passages the IOSO method outperforms the PGA method. Due to the larger number of design variables, both methods require more iterations and large number of simultaneous analyses per iteration.

Both IOSO and PGA methods reduced the number of passages from the initial design as shown in Table 6. However, one can see in Figure 10 that the topology of the passages is different between the two best designs. The result of the PGA is clearly a local optimum since some passages are clustered together near the trailing edge creating overcooled areas. A larger population size, more iterations, and more fine-tuning of control parameters could improve the PGA result.

Table 6: Optimization results for initially 30-passage case.

Result	Initial guess	PGA best design	IOSO best design
$n_{holes}$	30	23	24
$T_{max}$	763.2 °C	800.1 °C	800.6 °C
$F$	1132.6 W	1069.6 W	1059.3 W

The outer surface temperature on the optimized design is much closer to  $T_{allow}$  than in the initial design as shown in Figure 11. The best possible design could be achieved if the entire outer surface temperature would be equal to  $T_{allow}$ . In that case, the smallest possible integrated heat flux would be 1038.0 Watts. As in the case of initially 13 cooling passages, such a perfect design is not achievable due to the limitations of the geometric parameterization.

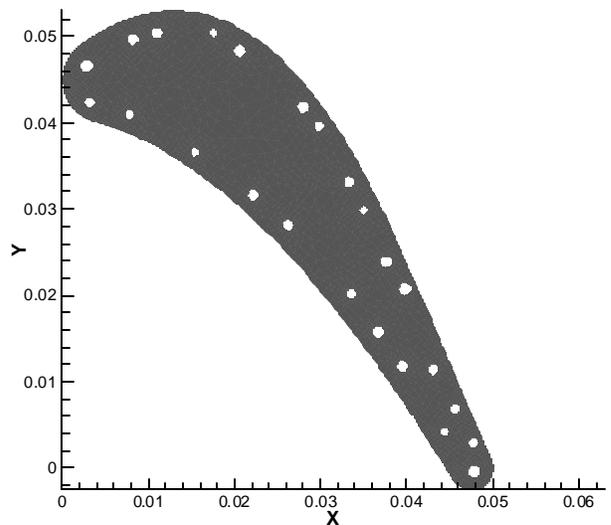
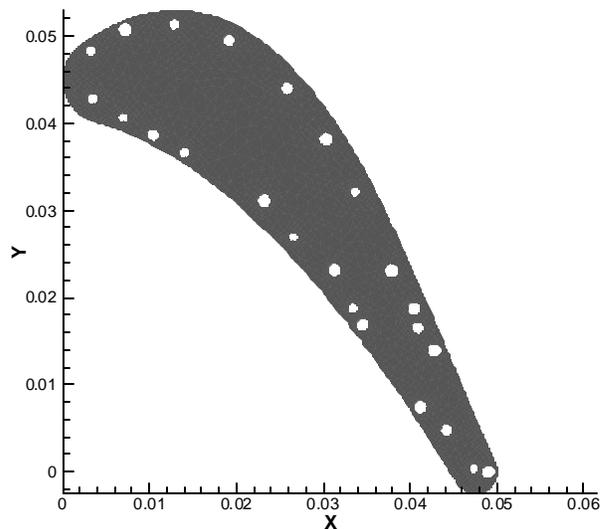
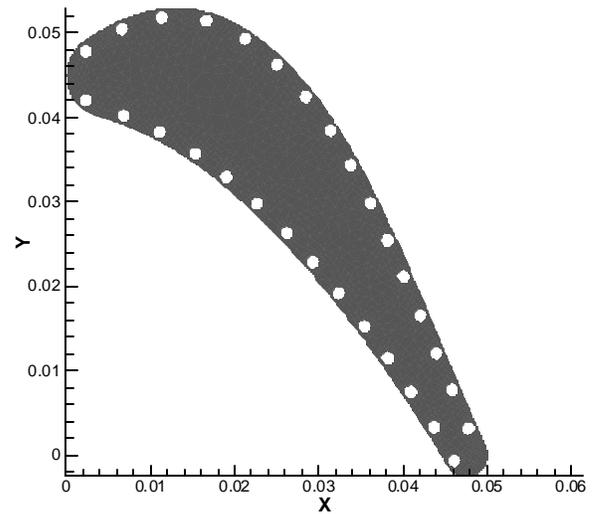


Figure 10: Cooling passages for initial design, PGA best design, and IOSO best design for initially 30-passage case.

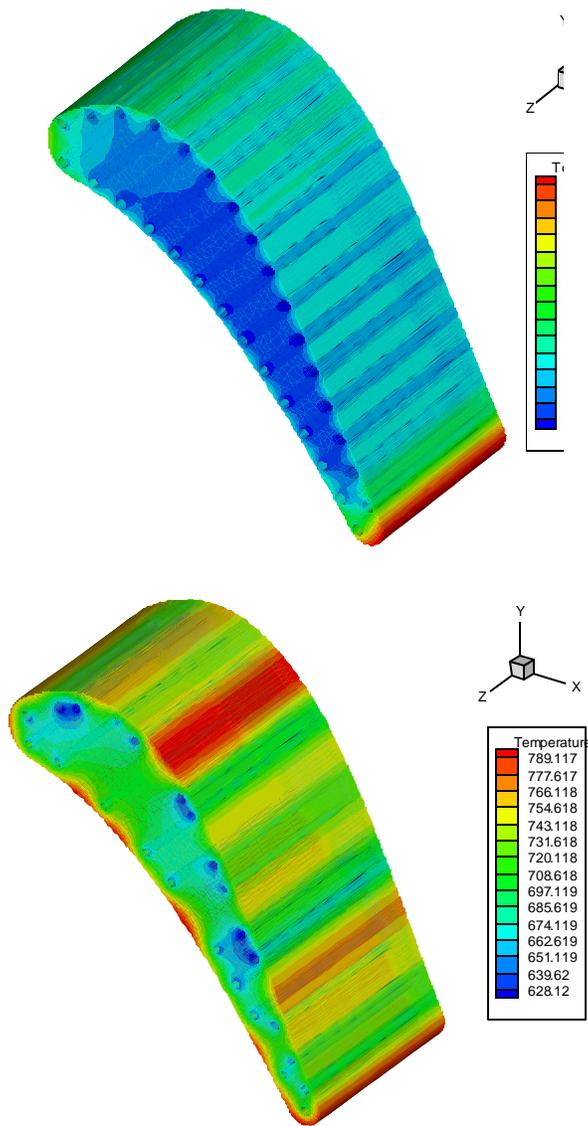


Figure 11: Temperature distribution for initial design and IOSO optimized best design for initially 30-passage case.

## CONCLUSIONS

A software system for the design of turbine blade coolant passages has been developed using powerful optimization algorithms and efficient parallel finite element thermal field analysis code. The automatic parametric shape design of many internal coolant passages was demonstrated. A typical design optimization can be completed within a few hours using an inexpensive cluster of personal computers. The IOSO optimization method was found to produce better results with fewer iterations than the PGA method. The IOSO method is also more robust and easier to use since it requires fewer tuning parameters than the PGA method.

The design optimization examples presented in this paper represent simplified cases as the effect of the inner and outer fluid mechanics is very approximate. Additional objective should have been used during the optimization that would

minimize temperature gradients along the hot outer surface. The next step towards a complete automatic design system should be to add 3-D fluid mechanics analysis codes and 3-D thermoelasticity analysis thus providing a fully 3-D conjugate analysis environment. However, this would then increase computing time by an order of magnitude. But, with the recent availability of low cost parallel supercomputing based on commodity component, a complete multidisciplinary design system may be proven to be computationally and financially feasible in the very near future.

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