

Robust Design Optimization Strategy of IOSO Technology

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Key words: optimization, robust design, response surface, multilevel optimization

Abstract

Robust Design Optimization is a high priority problem now. The concepts of Robust Design Optimization allow finding an optimal technical solution, accounting that such a technical solution could be implemented in practice with high probability.

The paper presents the main capabilities of the Robust Design Optimization Strategy of the IOSO Technology (Indirect Optimization based on Self-Organization). IOSO Technology implements the novel evolutionary response surface strategy. This strategy differs significantly from both the traditional approaches of nonlinear programming and the traditional response surface methodology.

The main feature of the Robust Design Optimization is that optimization problem is solved using stochastic formulation directly, when the evaluation of probability parameters is being performed at each iteration. High efficiency of the Robust Design Optimization is provided by the highly efficient capabilities of the developed stochastic optimization algorithms, which reliably work when high level of noise is present in responses.

The paper presents the assembly of the newly developed efficient approaches for solving problems requiring Robust Design Optimization. These approaches employ technology of multilevel and multiobjective optimization both separately and simultaneously.

The distinctive feature of the Multilevel Robust Design Optimization strategy is the use of mathematical models of various fidelity (from the lowest to the highest) during the solution process.

Developed algorithms of the Multiobjective Robust Design Optimization allow us to successfully solve the multiobjective (dozens of objective functions) multidimensional (hundreds of design variables) nonlinear optimization problems.

1 Introduction

Practical application of the numerical optimization results is complicated by the fact that any intricate technical system is a stochastic system and characteristics of this system have a probabilistic nature. This nature is in the presence of uncertainty elements for any real-life system. Generally, system operation is affected by uncertainties in geometrical parameters due to production technology spread; in materials properties; in mathematical model accuracy; in environment conditions, etc. The recent work by Batill, et.al [1] presents the number of issues related to the various types of uncertainty in multidisciplinary design optimization. In the present paper, speaking of stochastic properties of a technical system within the frame of optimization tasks, we imply a totality of system's essential parameters random spread. Random deviations of the system's parameters lead to a random change in system's efficiency.

In the current practice of deterministic optimization, uncertainties of design variables are either unaccounted for or accounted for by the safety factors. An efficiency extremum value, obtained during the optimization problem solving in a deterministic setting, is a maximum attainable value and can be considered as just conventional optimum from the point of view of its practical implementation. Based on this fact, Jacek Marczyk [1] states: "Optimization is actually just the opposite of robustness". He considers the stochastic simulation using the Monte-Carlo technique as an alternative to traditional optimization. Our experience in the field of real-life optimization allows us to reformulate Marczyk's thesis in the following form: "Deterministic optimization is actually just the opposite of robustness". Since the 1992 [2] we are speaking about the necessity to use the "stochastic optimization" vs. "deterministic optimization" for real-life problems.

In recent years, probabilistic design analysis and optimization methods have been developed to account for uncertainty and randomness through stochastic simulation and probabilistic analysis [2, 3, 5, 6, 8-10]. In our opinion totality of such methods can be treated as the new scientific direction, named "Robust Design Optimization" (RDO). The distinct feature of this direction is the use of probability criteria to evaluate the technical system quality. The current paper does not provide a full review of literature in this growing field, but does include selected papers, characterizing the most interesting features of this approach.

One dealing with RDO can consider two different types of optimization criteria (fig. 1). One of them is an ideal efficiency which can be achieved under the conditions of absolutely precise practical replication of the preset parameters of the system under consideration (deterministic criterion). Other optimization criteria are of probability nature. For example: mean value of the efficiency; total probability of assuring the preset constraints; variance of the efficiency and so on.

It is evident that the extremum of one of these criteria doesn't guarantee the assurance of the high level of another one. Even more, these criteria may be contradicting each other. Thus, in this case we have a multiobjective optimization problem as a rule.

Our concept of RDO allows determining the optimal practical technical solution that could be implemented with the high probability [2, 3, 5]. Many current stochastic approaches either employ estimation of probability efficiency criteria only at the stage of solution analysis [8, 10], or use an artificial approaches to estimate probability criteria during optimization process. The distinctive feature of our approach is that during robust design optimization we solve the optimization problem using direct stochastic formulation, when estimation of probabilistic criteria is being accomplished at each iteration. This procedure reliably produces truly robust optimal solution. We can solve RDO problems in this extensive formulation due to the unique robust properties of optimization algorithms which have been developed within the frames of IOSO technology. Moreover, we have developed a set of special algorithms aimed at time-consuming reduction of RDO problems solution.

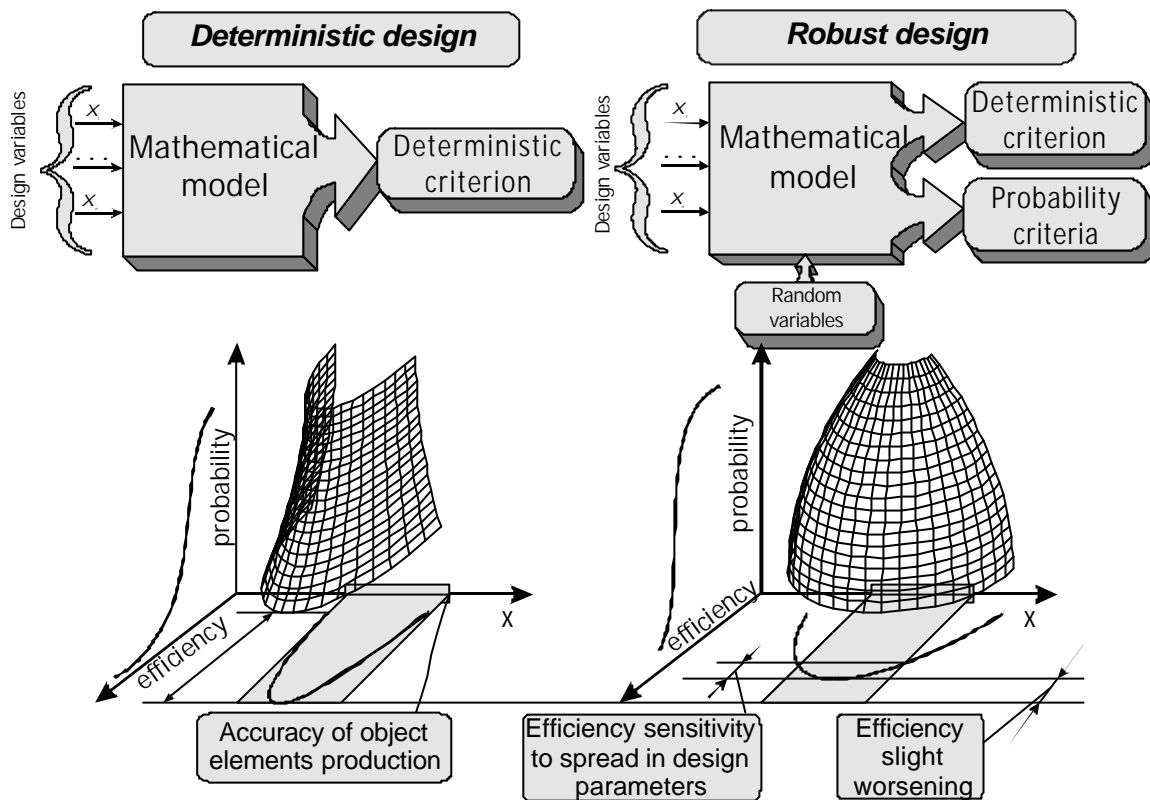


Fig.1 Robust design optimization essence.

2 The Main Features of the Robust Design Optimization Strategy

Suppose that efficiency of a system under study can be evaluated by indicator y which is to be minimized. This indicator can be presented as a function depending upon the vector of the deterministic design parameters $\bar{x} = (\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n)$, which is the vector of optimization variables, and upon the vector of random parameters \mathbf{x} . Vector of random parameters consists of two parties. First part can be interpreted as a random spread of design parameters in the real-life conditions, for example: $x_i = \bar{x}_i \times (1 + \mathbf{x}_i)$, $i = \overline{1, n}$. Second part is the random vector describing other uncertainties (environment, mathematical model accuracy and so on). Note, that when the components of vector \mathbf{x} are considered as deterministic, the problem is nothing but the deterministic optimization problem.

If there is a just enough practical experience of system production we can estimate the probability density functions of random vector components. Moreover, in most engineering cases these parameters have normal or close to normal distribution functions. Hence, in the frameworks of our approach we use the hypothesis that the distribution functions of \mathbf{x} - components are known.

One of the main problem of RDO is how to evaluate the probability criteria and constraints. In general, there are two approaches to probability indexes evaluation. One of them is the Monte Carlo simulation. This is a more general and reliable technique. But, the drawback of Monte Carlo simulation is the large number of sampling points usually required to obtain sufficiently accurate estimates of probability criteria and constraints. Second approach includes a number of methods which are based on different approximation techniques (Taylor's series, response surfaces and so on). An

examples of these methods one can see in the [10]. The approximate methods of probability indexes evaluation usually require an additional information from mathematical model such as: responses sensitivities; variable to variable dependencies etc. Moreover, these methods are very sensitive with respect to topological peculiarities of objective functions and constraints. Note, that any optimization algorithm realizes the iterative procedure with the large number of objective functions and constraints evaluations. Then, most of optimization algorithms (especially gradient-based algorithms) requires the sufficiently high accuracy of probability indexes evaluation. These leads to enormous time-consuming while RDO problem solving.

The peculiarity of IOSO RDO strategy is that it is not necessary to evaluate probability indexes highly accurately at each iteration of search. It is explained by noiseproof features of IOSO optimization algorithms. IOSO Technology implements the new evolutionary response surface methodology. This methodology differs significantly from both the traditional approaches of nonlinear programming and the traditional response surface approach. Because of that IOSO Technology algorithms have higher efficiency, provide wider range of capabilities, and are practically insensitive with respect to the types of objective function and constraints: smooth, non-differentiable, stochastic, with multiple optima, with the portions of the design space where objective function and constraints could not be evaluated at all, with the objective function and constraints dependent on mixed variables, etc. (see fig. 2).

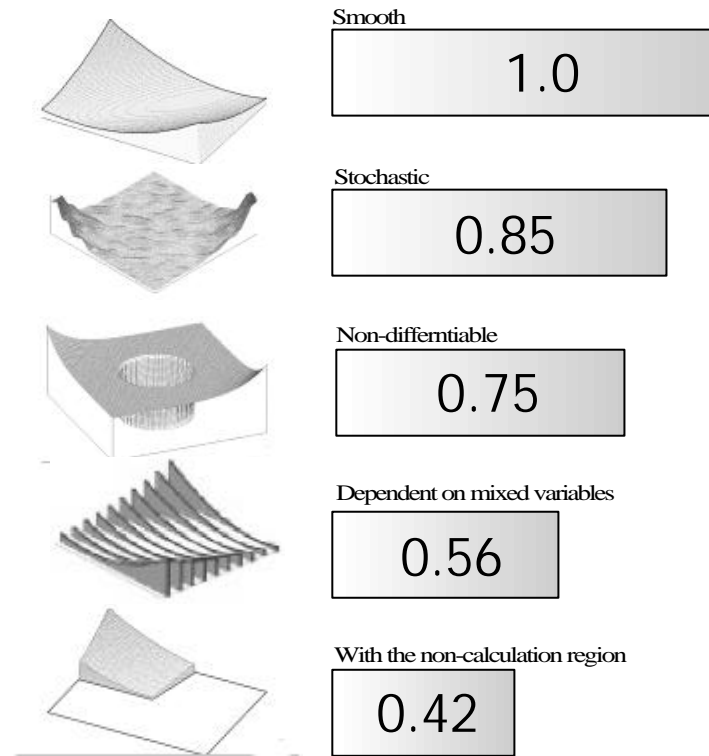


Fig. 2 IOSO algorithms efficiency for different objective functions.

So, we can use the Monte Carlo simulation with extremely low number of sampling points (tens or hundreds) while solving RDO problem, in order to reduce time-consuming. As we use the approximate estimations of probability criteria during the optimization, after the solution has been obtained one can carry out special statistical research to improve the final estimations. Within the frames of this general strategy we developed special RDO algorithms aimed at increasing of optimization efficiency in some particular cases. Let us briefly consider the main properties of these algorithms in the following sections of paper.

3 Multiobjective Robust Design Optimization

Usually, RDO problem is the problem of search for compromise between system's efficiency and reliability. Substantial choice of final design is possible only on the base of Pareto set analysis. To find the Pareto set one need to solve the multiobjective optimization problem. We developed the very effective algorithms of the Multiobjective Robust Design Optimization. The main advantages of the proposed algorithms over traditional mathematical programming approaches are the following [3]:

- convolution approaches are not used in solving multiobjective problems;
- the algorithms determine the desired number of Pareto-optimal solutions, so that these solutions are uniformly distributed in the space of objective functions;
- it is possible to solve the optimization problems for the objective functions of complex topology: non-convex, non-differentiable, with many local optima;
- relatively small number of probability indexes evaluations;
- it is possible to naturally employ the parallelization of the computational process.

These advantages are the basis for the wide use of the proposed method in the real-life problems. Let us consider the example of the multiobjective robust design optimization of the multistage axial flow compressor. The brief description of this optimization problem is given in the table 1.

Table 1. Brief description of the compressor robust design optimization problem.

Purpose	To insure the maximum efficiency and maximum implementation probability under preset level of production technology.
Setting features	140 independent variables (flow-path geometry); two objectives; three nonlinear constraints (mass flow, pressure ratio, surge margin).
Optimization process features	Object under study – quasi-3D mathematical model. Implementation probability was calculated as the probability of assuring the preset constraints.

Fig. 3 shows the main results of this problem. One can see that there is a compromise area between the ideal (deterministic) compressor efficiency and the implementation probability. In general, designer can select any solution from the obtained set. In this case the design No 4 was selected as the final design.

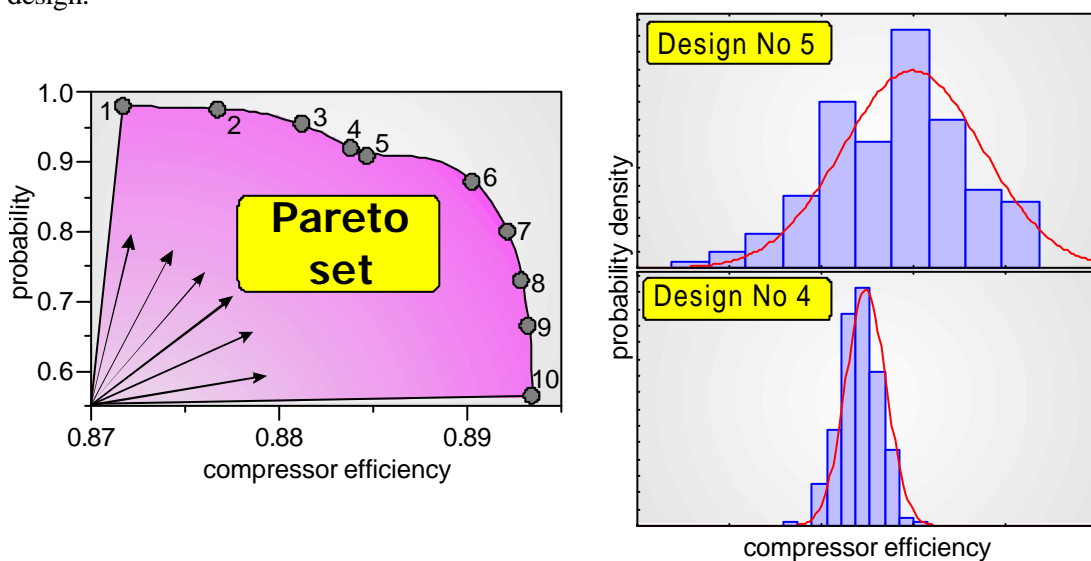


Fig. 3 Results of compressor multicriteria robust design optimization.

While solving this problem we used only 50 calls of mathematical model to approximately evaluate the probability criterion at each iteration. After 400 iterations optimization process was halted. Then we used 5000 additional calls of mathematical model to refine probability criteria for Pareto set found. Thus, total number of mathematical model calls was 25000. This is a not enormous value for the optimization problem having 140 independent variables, but solution of the same problem using more accurate mathematical model is problematic one. In this case reasonable choice is the usage of so-called multilevel RDO procedure.

4 Multilevel Robust Design Optimization

The feature of the Multilevel Robust Design Optimization procedure is the use of mathematical models of various fidelity (from the lowest to the highest) during the solution process and adaptive switching between them [4]. This procedure provides minimization of the number of times the high fidelity (true) models are used without reducing the accuracy of the resulting solution (fig. 4).

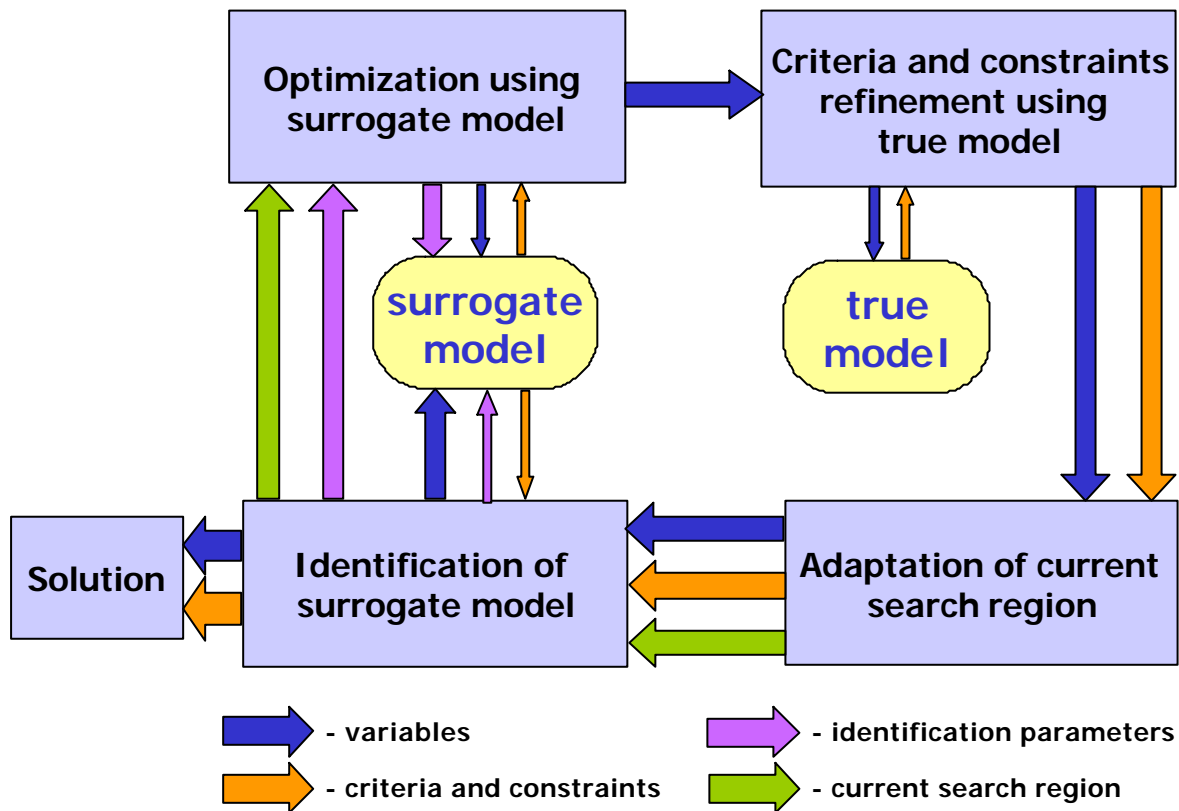


Fig. 4 Multilevel optimization scheme.

The efficiency of this procedure may be demonstrated using the similar compressor optimization problem with 63 independent variables (fig. 5). As the surrogate model we used the simplified axis-symmetrical model which is possible to be identified. The main idea of multilevel optimization is that information stored during the search is used to improve the surrogate model. However, this identification is correct not for the entire initial search area but only for certain neighborhood of the

obtained Pareto set. This ensures purposeful improvement of probability criteria evaluation accuracy only in the area of optimal solutions.

For this problem we obtained 10 Pareto-optimal solutions using only 60 direct calls to high-fidelity model. This example shows that it is possible to solve the optimization problem when the number of times the highest fidelity model is involved is less than the number of design variables. This provides considerable (several orders of magnitude) reduction in CPU time required for solution of complex optimization problems.

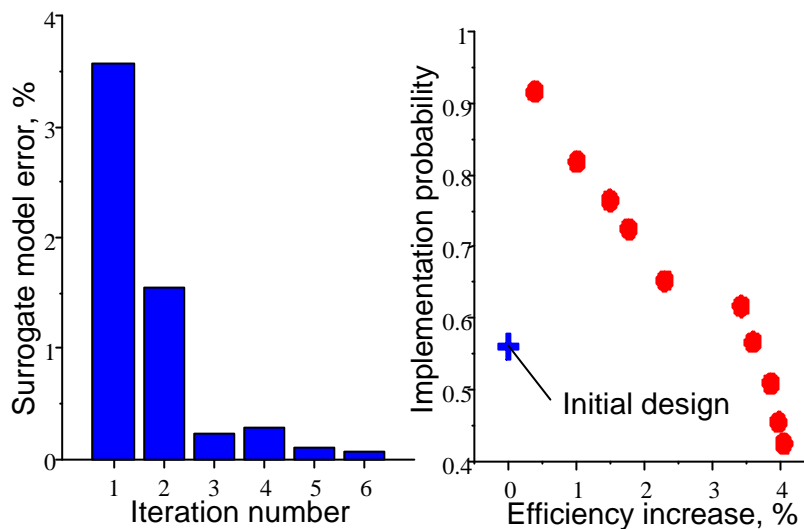


Fig. 5 Results of compressor multilevel robust design optimization.

5 CONCLUSION

Our experience in using the various Robust Design Optimization procedures applied to test problems and real-life problems shows that the total efficiency of the optimization process could be increased 5-10 times. This indicates that combining these procedures significantly broadens the capabilities of the Robust Design Optimization strategy when applied to real-life systems. Using our Robust Design Optimization concept results in considerable (several orders of magnitude) cost and time reduction when developing new highly efficient technical systems. Using Robust Design Optimization concept also provides the considerable (several times) risk reduction when new technical solutions are implemented.

References

- [1] S. M. Batill, J. E. Renaud and Xiaoyu Gu, *Modeling and Simulation Uncertainty in Multidisciplinary Design Optimization*, AIAA-2000-4803, Proceedings of 8th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Long Beach, USA, (2000).

- [2] N. Egorov, *Optimization of a Multistage Axial Compressor. Stochastic Approach*, ASME, 92-GT-163, (1992).
- [3] N. Egorov and G. V. Kretinin, *Search for Compromise Solution of the Multistage Axial Compressor's Stochastic Optimization Problem*, World Publishing Corporation, *Aerothermodynamics of internal flows III*, Beijing, China, (1996), pp. 112-120.
- [4] N. Egorov, G. V. Kretinin, I. A. Leshchenko, Y. I. Babiy, *Optimization of complex engineering systems using variable-fidelity models*, MCB University Press, ISBN: 0-86176-650-4, Proceedings of the 1st ASMO UK/ISSMO conference on Engineering Design Optimization, (1999), pp.143-149.
- [5] N. Egorov, G. V. Kretinin, I. A. Leshchenko, *Stochastic Optimization of Parameters and Control Laws of the Aircraft Gas-Turbine Engines – a Step to a Robust Design*, Elsevier Science Ltd, “Inverse Problem in Engineering Mechanics III”, (2002), pp.345...353.
- [6] P. N. Koch, B. Wujek and O. Golovidov, A Multi-Stage, *Parallel Implementation of Probabilistic Design Optimization in an MDO Framework*, AIAA-2000-4805, Proceeding of 8th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Long Beach, USA, (2000).
- [7] Jacek Marczyk, *Stochastic Multidisciplinary Improvement: Beyond Optimization*, AIAA-2000-4929, Proceedings of 8th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Long Beach, USA, (2000).
- [8] M. T. Tong, *A Probabilistic Approach to Aeropropulsion System Assessment*, 2000-GT-0001, Proceedings of ASME TURBOEXPO 2000, Munich, Germany, (2000).
- [9] Ryohei Yokoyama and Koichi Ito, *Robust Optimal Design of a Gas Turbine Cogeneration Plant Based on Minimax Regret Criterion*, 99-GT-128, Presented at the International Gas Turbine & Aeroengine Congress & Exhibition, Indianapolis, USA, (1999).
- [10] Y-T. Wu, *Methods for Efficient Probabilistic Analysis of Systems with Large Numbers of Random Variables*, AIAA-98-4908, Proceeding of 7th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, St. Louis, USA, (1998).